(25) $\frac{1}{1-\cos C}\left(\frac{1+\cos C}{1+\cos C}\right)+\frac{1}{1+\cos C}\left(\frac{1-\cos C}{1-\cos C}\right)$

$= \frac{1+\cos C+1-\cos C}{1-\cos^2 C}$

$= \frac{2}{\sin^2 C}$

$= \frac{2}{1}\left(\frac{1}{\sin^2 C}\right)$

$= 2\csc^2 C$

(26) $\frac{1}{\sec D-\tan D}\left(\frac{\sec D+\tan D}{\sec D+\tan D}\right)+\frac{1}{\sec D+\tan D}\left(\frac{\sec D-\tan D}{\sec D-\tan D}\right)$

$= \frac{\sec D+\tan D+\sec D-\tan D}{\sec^2 D-\tan^2 D}$

$= \frac{2\sec D}{1}$

$= 2\sec D$

(20) $\frac{1-\cos^2 x}{\tan x}$

$= \frac{\sin^2 x}{\tan x}$

$= \sin^2 x \left(\frac{\cos x}{\sin x}\right)$

$= \sin x \cos x$

$\therefore \frac{1-\cos^2 x}{\tan x} = \sin x \cos x$

(21) $\frac{\sec^2\theta - 1}{\sin\theta}$

$= \frac{\tan^2\theta}{\sin\theta}$

$= \frac{\sin^2\theta}{\cos^2\theta}\left(\frac{1}{\sin\theta}\right)$

$= \frac{\sin\theta}{\cos\theta}\left(\frac{1}{\cos\theta}\right)$

$= \tan\theta \sec\theta$

$\therefore \frac{\sec^2\theta - 1}{\sin\theta} = \tan\theta \sec\theta$

(22) $\frac{1+\cot^2\theta}{\sec^2\theta}$

$= \frac{\csc^2\theta}{\sec^2\theta}$

$= \frac{1}{\sin^2\theta} \cdot \frac{\cos^2\theta}{1}$

$= \frac{\cos^2\theta}{\sin^2\theta} = \cot^2\theta$

$\therefore \frac{1+\cot^2\theta}{\sec^2\theta} = \cot^2\theta$

(23) $\left(\frac{\cos A}{\cos A}\right)\frac{\sec A}{\sin A} - \frac{\sin A}{\cos A}\left(\frac{\sin A}{\sin A}\right)$

$= \frac{1-\sin^2 A}{\cos A \sin A}$

$= \frac{\cos^2 A}{\cos A \sin A}$

$= \frac{\cos A}{\cos A}\left(\frac{\cos A}{\sin A}\right)$

$= \cot A$

$\therefore \frac{\sec A}{\sin A} - \frac{\sin^2 A}{\cos A} = \cot A$

(24) $\frac{\csc B}{\cos B} - \frac{\cos B}{\sin B}$

$\frac{\sin B \csc B}{(\sin B)(\cos B)} - \frac{\cos B}{\sin B}\left(\frac{\cos B}{\cos B}\right)$

$= \frac{1-\cos^2 B}{\sin B \cos B}$

$= \frac{\sin^2 B}{\sin B \cos B}$

$= \frac{\sin B}{\sin B}\left(\frac{\sin B}{\cos B}\right)$

$= \tan B$

$\frac{\csc B}{\cos B} - \frac{\cos B}{\sin B} = \tan B$

(15) $(\cos\phi - \sin\phi)^2$

$= \cos^2\phi - 2\cos\phi\sin\phi + \sin^2\phi$

$= 1 - 2\cos\phi\sin\phi$

$\therefore (\cos\phi - \sin\phi)^2 = 1 - 2\cos\phi\sin\phi$

(16) $(1 - \tan\phi)^2$

$= 1 - 2\tan\phi + \tan^2\phi$

$= \sec^2\phi - 2\tan\phi$

$\therefore (1 - \tan\phi)^2 = \sec^2\phi - 2\tan\phi$

(17) $(\tan n + \cot n)^2$ $\quad 1$

$= \tan^2 n + 2(\cot n \tan n) + \cot^2 n$

$= \tan^2 n + 2 + \cot^2 n$

$= (\sec^2 n - 1) + 2 + (\csc^2 n - 1)$

$= \sec^2 n + \csc^2 n$

$\therefore (\tan n + \cot n)^2 = \sec^2 n + \csc^2 n$

(18) $(\cos K - \sec K)^2$ $\quad 1$

$= \cos^2 K - 2(\cos K \sec K) + \sec^2 K$

$= \cos^2 K - 2 + \sec^2 K$

$= (1 - \sin^2 K) - 2 + (\tan^2 K + 1)$

$= \tan^2 K - \sin^2 K$

$\therefore (\cos K - \sec K)^2 = \tan^2 K - \sin^2 K$

(19) $\dfrac{\csc^2 x - 1}{\cos x}$

$= \dfrac{\cot^2 x}{\cos x}$

$= \dfrac{\cos^2 x}{\sin^2 x}\left(\dfrac{1}{(\cos x)}\right)$ $\quad \cos x$ $\quad 1$

$= \dfrac{\cos x}{\sin x}\left(\dfrac{1}{\sin x}\right)$

$= \cot x \csc x$

$\therefore \dfrac{\csc^2 x - 1}{\cos x} = \cot x \csc x$

(8) $\sin\theta + \cot\theta\cos\theta$

$= \frac{\sin\theta}{1}\left(\frac{\sin\theta}{\sin\theta}\right) + \frac{\cos\theta}{\sin\theta}\left(\frac{\cos\theta}{1}\right)$

$= \frac{\sin^2\theta + \cos^2\theta}{\sin\theta}$

$= \frac{1}{\sin\theta}$

$= \csc\theta$

$\therefore \sin\theta + \cot\theta\cos\theta = \csc\theta$

(9) $\csc x - \sin x$

$= \frac{1}{\sin x}\left(\frac{1}{1}\right) - \frac{\sin x}{1}\left(\frac{\sin x}{\sin x}\right)$

$= \frac{1 - \sin^2 x}{\sin x}$

$= \frac{\cos^2 x}{\sin x}$

$= \frac{\cos x}{\sin x}\left(\frac{\cos x}{1}\right)$

$= \cot x \cos x$

$\therefore \csc x - \sin x = \cot x \cos x$

(10) $\sec\theta - \cos\theta$

$= \frac{1}{\cos\theta} - \frac{\cos\theta}{1}\left(\frac{\cos\theta}{\cos\theta}\right)$

$= \frac{1 - \cos^2\theta}{\cos\theta}$

$= \frac{\sin^2\theta}{\cos\theta}$

$= \frac{\sin\theta}{\cos\theta}\left(\frac{\sin\theta}{1}\right)$

$= \tan\theta \cdot \sin\theta$

(11) $\tan x(\sin x + \cot x \cos x)$

$= \tan x \sin x + \tan x \cot x \cos x$

$= \left(\frac{\sin x}{\cos x}\right)\frac{\sin x}{1} + \cos x\left(\frac{\cos x}{\cos x}\right)$

$= \frac{\sin^2 x}{\cos x} + \frac{\cos^2 x}{\cos x}$

$= \frac{1}{\cos x}$

$= \sec x$

$\therefore \tan x(\sin x + \cot x \cos x) = \sec x$

(12) $\cos x(\sec x + \cos x \csc^2 x)$

$= \cos x \sec x + \cos^2 x \csc^2 x$

$= 1 + \frac{\cos^2 x}{\sin^2 x}$

$= 1 + \cot^2 x$

$= \csc^2 x$

$\therefore \cos(\sec x + \cos x \csc^2 x) = \csc^2 x$

(13) $(1+\sin B)(1-\sin B)$

$= 1 - \sin^2 B$

$= \cos^2 B$

$\therefore (1+\sin B)(1-\sin B) = \cos^2 B$

(14) $(\sec x - 1)(\sec x + 1)$

$= \sec^2 x - 1$

$= \tan^2 x$

$\therefore (\sec x - 1)(\sec x + 1) = \tan^2 x$

① $\cos x \tan x$

$= \cancel{\cos x}\left(\frac{\sin x}{\cancel{\cos x}_1}\right)$

$= \sin x$

$\therefore \cos x \tan x = \sin x$

② $\csc x \tan x$

$= \frac{1}{\cancel{\sin x}_1}\left(\frac{\cancel{\sin x}^1}{\cos x}\right)$

$= \frac{1}{\cos x}$

$= \sec x$

$\therefore \csc x \tan x = \sec x$

③ $\sec x \cot x \sin x$

$= \frac{1}{\cancel{\cos x}_1}\left(\frac{\cancel{\cos x}^1}{\cancel{\sin x}_1}\right)\left(\frac{\cancel{\sin x}^1}{1}\right)$

$= 1$

$\therefore \sec x \cot x \sin x = 1$

④ $\csc x \tan x \cos x$

$= \frac{1}{\cancel{\sin x}_1}\left(\frac{\cancel{\sin x}^1}{\cos x}\right)\left(\frac{\cancel{\cos x}^1}{1}\right)$

$= 1$

$\therefore \csc x \tan x \cos x = 1$

⑤ $\sin^2\theta \sec\theta \csc\theta$

$\sin^2\theta \; \frac{\cancel{\sin^2\theta}}{1}\left(\frac{1}{\cos\theta}\right)\left(\frac{1}{\cancel{\sin\theta}_1}\right)$

$= \frac{\sin\theta}{\cos\theta}$

$= \tan\theta$

$\therefore \sin^2\theta \sec\theta \csc\theta = \tan\theta$

⑥ $\cos^2 A \csc A \sec A$

$\cos A \; \frac{\cancel{\cos^2 A}}{1}\left(\frac{1}{\sin A}\right)\left(\frac{1}{\cancel{\cos A}_1}\right)$

$= \frac{\cos A}{\sin A}$

$= \cot A$

$\therefore \cos^2 A \csc A \sec A = \cot A$

⑦ $\tan A + \cot A$

$= \frac{\sin A}{\cos A}\left(\frac{\sin A}{\sin A}\right) + \frac{\cos A}{\sin A}\left(\frac{\cos A}{\cos A}\right)$

$= \frac{\sin^2 A + \cos^2 A}{\sin A \cos A}$

$= \frac{1}{\sin A \cos A}$

$= \frac{1}{\sin A}\left(\frac{1}{\cos A}\right)$

$= \csc A \sec A$

$\therefore \tan A + \cot A = \csc A \sec A$