① $\sec x(\sec x - \cos x) = \tan^2 x$

$= \sec^2 x - \sec x \cos x)$

$= \sec^2 x - 1$

$= \tan^2 x$

$\therefore \sec x(\sec x - \cos x) = \tan^2 x$

② $\tan x(\cot x + \tan x) = \sec^2 x$

$= \tan x \cot x + \tan^2 x$

$= 1 + \tan^2 x$

$= \sec^2 x$

$\therefore \tan x(\cot x + \tan x) = \sec^2 x$

③ $\sin x(\csc x - \sin x) = \cos^2 x$

$= \sin x \csc x - \sin^2 x$

$= 1 - \sin^2 x$

$= \cos^2 x$

$\therefore \sin x(\csc x - \sin x) = \cos^2 x$

④ $\cos x(\sec x - \cos x) = \sin^2 x$

$= \cos x(\sec x) - \cos^2 x$

$= 1 - \cos^2 x$

$= \sin^2 x$

$\therefore \cos x(\sec x - \cos x) = \sin^2 x$

⑤ $\csc^2 \theta - \cos^2 \theta \csc^2 \theta = 1$

$= \csc^2 \theta(1 - \cos^2 \theta)$

$= \csc^2 \theta(\sin^2 \theta)$

$= 1$

(6) $\cos^2\theta + \tan^2\theta \cos^2\theta = 1$

$= \cos^2\theta + \left(\frac{\sin^2\theta}{\cos^2\theta}\right)\frac{\cos^2\theta}{1}$

$= \cos^2\theta + \sin^2\theta$

$= 1$

(7) $(\sec\theta + 1)(\sec\theta - 1) = \tan^2\theta$

$= \sec^2\theta - 1$

$= \tan^2\theta$

(8) $(1 + \sin\theta)(1 - \sin\theta) = \cos^2\theta$

$= 1 - \sin^2\theta$

$= \cos^2\theta$

(9) $\sec^2 A + \tan^2 A \sec^2 A = \sec^4 A$

$= \sec^2 A (1 + \tan^2 A)$

$= \sec^2 A (\sec^2 A)$

$= \sec^4 A$

(10) $\cot^2 A \csc^2 A - \cot^2 A = \cot^4 A$

$= \cot^2 A (\csc^2 A - 1)$

$= \cot^2 A (\cot^2 A)$

$= \cot^4 A$

11. $\cos^4 t - \sin^4 t = 1 - 2\sin^2 t$

$= (\cos^2 t - \sin^2 t)(\cos^2 t + \sin^2 t)$

$= (\cos^2 t - \sin^2 t)(1)$

$= (1 - \sin^2 t - \sin^2 t)$

$= 1 - 2\sin^2 t$

12. $\sec^4 t - \tan^4 t = 1 + 2\tan^2 t$

$= (\sec^2 t - \tan^2 t)(\sec^2 t + \tan^2 t)$

$= (1)(\sec^2 t + \tan^2 t)$

$= 1 + \tan^2 t + \tan^2 t$

$= 1 + 2\tan^2 t$

(13) $\dfrac{1}{\sin x \cos x} - \dfrac{\cos x}{\sin x}\left(\dfrac{\cos x}{\cos x}\right) = \cot x$

$= \dfrac{1 - \cos^2 x}{\sin x \cos x}$

$= \dfrac{\sin^2 x \sin x}{\sin x \cos x}$

$= \cot x$

(14) $\dfrac{\sec x}{\sin x}\left(\dfrac{\cos x}{\cos x}\right) - \dfrac{\sin x}{\cos x}\left(\dfrac{\sin x}{\sin x}\right) = \cot x$

$= \dfrac{(\sec x)(\cos x) - \sin^2 x}{\sin x \cos x}$

$= \dfrac{1 - \sin^2 x}{\sin x \cos x}$

$= \dfrac{\cos^2 x}{\sin x \cos x} = \cot x$

(15) $\frac{\sin x}{\csc x} + \frac{\cos x}{\sec x} = 1$

$= \frac{\sin x}{1}\left(\frac{1}{\csc x}\right) + \frac{\cos x}{1}\left(\frac{1}{\sec x}\right)$

$= \sin x(\sin x) + \cos x(\cos x)$

$= \sin^2 x + \cos^2 x$

$= 1$

(16) $\frac{1}{\sec^2 x} + \frac{1}{\csc^2 x} = 1$

$= \cos^2 x + \sin^2 x$

$= 1$

(17) $\frac{1}{1+\cos s}\left(\frac{1-\cos s}{1-\cos s}\right) = \csc^2 s - \csc s \cot s$

$= \frac{1-\cos s}{1-\cos^2 s}$

$= \frac{1-\cos s}{\sin^2 s}$

$= \frac{1}{\sin^2 s} - \frac{\cos s}{\sin^2 s}$

$= \csc^2 s - \csc s \cot s$

(18) $\frac{1}{1-\sin r}\left(\frac{1+\sin r}{1+\sin r}\right) = \sec^2 r + \sec r \tan r$

$= \frac{1+\sin r}{1-\sin^2 r}$

$= \frac{1-\sin r}{\cos^2 r}$

$= \frac{1}{\cos^2 r} - \frac{\sin r}{\cos^2 r}$

$= \sec^2 r - \frac{\sin r}{\cos r}\left(\frac{1}{\cos r}\right)$

$= \sec^2 r - \tan r \sec r$

(19) $\frac{\cos x}{\sec x - 1}\left(\frac{\sec x + 1}{\sec x + 1}\right) - \frac{\cos x}{\tan^2 x} = \cot^2 x$

$= \frac{\cos(\sec x) + \cos x}{\sec^2 x - 1} - \frac{\cos x}{\tan^2 x}$

$= \frac{1 + \cos x}{\tan^2 x} - \frac{\cos x}{\tan^2 x}$

$= \frac{1}{\tan^2 x}$

$= \cot^2 x$

(20)

$$\frac{\sin x}{1-\cos x}\left(\frac{1+\cos x}{1+\cos x}\right)+\frac{1-\cos x}{\sin x} = 2\csc x$$

$$= \frac{\sin x + \sin x\cos x}{1-\cos^2 x} + \frac{1-\cos x}{\sin x}$$

$$= \frac{\sin^1\!\!\not{x} + \sin\!\!\not{x}\cos x}{\sin^{\not 2}\!\!\not{x}_{\sin x}} + \frac{1-\cos x}{\sin x}$$

$$= \frac{1 + \cancel{\cos x} + 1 - \cancel{\cos x}}{\sin x}$$

$$= \frac{2}{\sin x}$$

$$= 2\csc x$$

(21) $$\frac{\sec x}{\sec x - \tan x}\left(\frac{\sec x + \tan x}{\sec x + \tan x}\right) = \sec^2 x + \sec x \tan x$$

$$= \frac{\sec^2 x + \sec x \tan x}{\sec^2 x - \tan^2 x} \quad (\sec^2 x - \tan^2 x = 1)$$

$$= \sec^2 x + \sec x \tan x$$

(22) $\frac{1+\sin x}{1-\sin x}\left(\frac{1+\sin x}{1+\sin x}\right) = 2\sec^2 x + 2\sec x\tan x - 1$

$= \frac{1+2\sin x+\sin^2 x}{1-\sin^2 x}$

$= \frac{1+2\sin x+\sin^2 x}{\cos^2 x}$

$= \frac{1}{\cos^2 x} + \frac{2\sin x}{\cos x}\left(\frac{1}{\cos x}\right) + \frac{\sin^2 x}{\cos^2 x}$

$= \sec^2 x + 2\sec x\tan x + \tan^2 x$

$= \sec^2 x + 2\sec x\tan x + \sec^2 x - 1$

$= 2\sec^2 x + 2\sec x\tan x - 1$

(23) $\sin^3 z\cos^2 z = \sin^3 z - \sin^5 z$

$= \sin^5 z\,(1-\sin^2 z)$

$= \sin^3 z - \sin^5 z$

(24) $\sin^3 z\cos^2 z = \cos^2 z\sin z - \cos^4 z\sin z$

$= \sin z(\sin^2)(\cos^2 z)$

$= \sin z(1-\cos^2 z)(\cos^2 z)$

$= (\sin z - \sin z\cos^2 z)(\cos^2 z)$

$= \cos z\sin z - \cos^4 z\sin z$

(25) $\sec^2\theta + \csc^2\theta = \sec^2\theta \csc^2\theta$

$\left(\frac{\sin^2\theta}{\sin^2\theta}\right)\frac{1}{\cos^2\theta} + \frac{1}{\sin^2\theta}\left(\frac{\cos^2\theta}{\cos^2\theta}\right)$

$= \frac{\sin^2\theta + \cos^2\theta}{\cos^2\theta \sin^2\theta}$

$= \frac{1}{\cos^2\theta \sin^2\theta}$

$= \sec^2\theta \csc^2\theta$

(26) $\sec\theta + \tan\theta = \frac{1}{\sec\theta - \tan\theta}\left(\frac{\sec\theta + \tan\theta}{\sec\theta + \tan\theta}\right)$

$= \frac{\sec\theta + \tan\theta}{\sec^2 - \tan^2\theta}$

$= \frac{\sec\theta + \tan\theta}{1}$

(27) $\frac{1 - 3\cos x - 4\cos^2 x}{\sin^2 x} = \frac{1 - 4\cos x}{1 - \cos x}\left(\frac{1+\cos x}{1+\cos x}\right)$

$= \frac{1 - 3\cos x - 4\cos^2 x}{1 - \cos^2 x}$

$= \frac{1 - 3\cos x - 4\cos^2 x}{\sin^2 x}$

(28) $\frac{\sec^2 x - 6\tan x + 7}{\sec^2 x - 5} = \frac{\tan x - 4}{\tan x + 2}\left(\frac{\tan x - 2}{\tan x - 2}\right)$

[illegible]

$= \frac{\tan^2 x - 6\tan x + 8}{\tan^2 x - 4}$

$$= \frac{\sec^2 x - 1 - 6 + \tan x + 8}{\sec^2 x - 1 - 4}$$

$$= \frac{\sec^2 x - 6 + \tan x + 7}{\sec^2 x - 5}$$

(29) $\dfrac{\sin^3 A + \cos^3 A}{\sin A + \cos A} = 1 - \sin A \cos A$ ← Sum of two cubes

$$\frac{(\sin A + \cos A)(\sin^2 A - \sin A \cos A + \cos^2 A)}{\sin A + \cos A}$$

$$= \sin^2 A - \sin A \cos A + \cos^2 A$$

$$= 1 - \sin A \cos A$$

(30) $\dfrac{\sec^2 B - \cos^3 B}{\sec B - \cos B} = \sec^2 B + 1 + \cos^2 B$

$$\frac{(\sec B - \cos B)(\sec^2 B + \sec B \cos B + \cos^2 B)}{\sec B - \cos B}$$

$= 1$

$$= \sec^2 B + \sec B \cos B + \cos^2 B$$

$$= \sec^2 B + 1 + \cos^2 B$$